Factorization Properties of Puiseux Monoids

Mao, Jason (School: The Academy for Mathematics, Science &amp
Engineering)

As factorization theory of rings is already well-studied, it is natural to generalize the scope of factorization theory from rings to monoids. Puiseux monoids (i.e. additive monoids over a subset of the positive rationals) fundamentally serve as a simple context under which crucial factorization properties such as the ACCP (ascending chain condition of principal ideals) and atomicity conditions in factorization theory may be analyzed. This paper accomplishes two goals in the study of factorization in Puiseux monoids: we prove that the nearly ACCP condition does not extend from Puiseux monoids to their respective monoid algebras, and we explore various examples and theorems regarding the Betti elements and Betti graphs of various Puiseux monoids. The former result is surprising, as many conditions such as the ACCP condition have been observed to extend from Puiseux monoids to their respective monoid algebras, whereas we present a counterexample to such an extension in the case of the nearly ACCP condition. We also explore Betti elements and Betti graphs through a variety of angles, touching upon the notions of atomization and connected components of Betti graphs. These results may help guide future research of factorization of Puiseux monoids and their respective monoid algebras, while also providing a clearer and more intuitive understanding of Betti elements and Betti graphs of Puiseux monoids. Ultimately, a broadened understanding of factorization properties and objects in Puiseux monoids may inspire future progress in the field of factorization theory of monoids as a whole.

Awards Won:
Mu Alpha Theta, National High School and Two-Year College Mathematics Honor Society: Second Award of $1,000