Along the Path of the Great Kaprekar: A-Function, Repunits and Their Properties

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Inspired by the famous function of the D.Kaprekar K(n)=n+S(n), I invented and investigated various properties of a new function A(n)=(n-S(n))/9 where S(n) is the sum of digits of the natural number n in decimal notation. The values of the function A(n) are called a-generated numbers and for the given value m=A(n) the argument n is called its a-generator. If a number p has no a-generators, then p is called an a-self number. The rank r(n)=u of a number n is the smallest number u such that A(A(…A(n)))= A^((u))(n)=0. Repunits are numbers whose decimal notation consists of only units. The connection between A(n) and repunits is established. Two new formulas for the class of repunit numbers are proved. Non-decreasing of the function A(n) and its other properties are proved. It is established that every a-generated number has exactly 10 a-generators. The infinity of a-self numbers is proved. It is shown, that fraction of a-self numbers within all numbers with given number of digits is one tenth. Formulas for "neighboring" (sequence of consecutive) a-self numbers with sequence lengths 1 and 2 are found. A pattern and a hypothesis for the distribution of "neighboring" a-self numbers with any other sequence lengths are formulated. The equations K(n)=q×A(n) and A(qn)=q×A(n) are completely solved. It is proved that the function r(n) is non-decreasing.

Awards Won:
King Abdulaziz &amp
his Companions Foundation for Giftedness and Creativity: Full Scholarship from King Fahd University of Petroleum and Minerals(KFUPM) (and a $400 cash prize)
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