# Expected Dispatches of Firefighting Helicopters Under the Optimal Strategy

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This project investigates a scenario in which a group of houses is on fire, prompting the fire department to deploy helicopters. Each dispatch extinguishes the target house's fire, with each adjacent house having a 50% chance of also being extinguished. The goal is to determine the optimal helicopter positions to minimize the expected number of dispatches required to extinguish the fire and calculate this expected number. We explore four house arrangements: 1. A linear arrangement of n houses. 2. A circular arrangement of n houses. 3. A T-shaped arrangement with j and k houses connected to two central houses. 4. A circular arrangement of n houses attached to a linear tail of m houses. By analyzing the graphical structure of each arrangement using recurrence relations, complex numbers, and characteristic polynomials, we determine the best dispatching strategies and compute the expected number of dispatches. For the linear arrangement, the optimal firefighting positions are independent of the number of houses. This strategy is then applied to the circular and T-shaped arrangements. The results for the circular arrangement with a linear tail are particularly interesting. The optimal strategy varies based on the values of n and m depending on whether n = 6 or n = 5. Finally, we derive the generating functions for the expected number of dispatches under the optimal strategies for each arrangement, thereby fully solving the four scenarios described above.

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