Exploration of the Ninja Path

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This study is from 2023 IMO P5: “A Japanese triangle consists of 1+2+...+n circles arranged in an equilateral triangular shape, with each i th row containing exactly i circles, one of which is colored red. A ninja path is a sequence of n circles forming a path from the top to the bottom row, with each step moving to one of the two adjacent circles below. In terms of n, find the greatest k such that in each Japanese triangle there is a ninja path containing at least k red circles.” We determine the value of k in the original problem and identify possible patterns to achieve the maximum k. We observe that different values of n may correspond to the same values of k, and k increases monotonically. Therefore, we only need to consider the minimum value of n required for each k. We then extend the problem by varying the number of balls per row and obtaining optimal patterns. The research is then generalized to three and higher dimensional hypercube. By optimizing the construction, we derive a solution for the minimization problem. Our main result establishes that in a D-dimensional space where each layer contains m balls, the minimum number of layers n required to ensure that a ninja path passes through at least k red balls is asymptotic to (m+1)/m ?^(D-1) k, where ? represents Knuth’s up-arrow notation. Additionally, we explore the total number of solutions for certain special cases, further deepening our understanding of the problem's structure.

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