The Smallest Regular (n+1)-gon That Covers a Regular n-gon With Constant Side Length and Shares a Vertex With It

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Polygons, a fundamental concept within the sub-learning area of geometry, have a versatile structure that makes them highly suitable for generalization. With applications extending across mathematics, engineering, and STEAM fields, polygons play a critical role in solving practical problems. This study focuses on determining the smallest regular (n+1)-gon that can completely cover a regular n-gon with a constant side length while sharing a vertex. The research is inspired by a puzzle from the “Brain Teasers” section of the TUBITAK Science and Technology magazine, which poses the question: “What is the area of the smallest square that can completely cover an equilateral triangle with sides measuring 1 unit?” This study begins with preliminary research on the geometric properties of equilateral triangles and squares, which is then generalized to determine the area of the smallest regular (n+1)-gon that can cover a regular n-gon while sharing a vertex. The conditions required to minimize areas when two polygons share a vertex have been analyzed and demonstrated. The relationship between the acute angle formed by the edges of two regular polygons and their areas has also been thoroughly examined. This study lays the groundwork for understanding the geometric relationships between regular polygons and their optimal arrangements. By extending the findings to practical applications, such as optimizing the placement of an equilateral triangle within a square container, this research demonstrates its potential to address real-world challenges in different fields.

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