Squarefullness of Factorial Products of Polynomials

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For a polynomial P(x) with integer coefficients and a maximum positive integer root m, we consider the product of its values from x=m+1 to n and denote it as Fp(n). Building up on previous work on irreducible quadratics and x^l +- q^l when l is odd, we prove that Fp(n) is not squarefull for sufficiently large n, when P(x) = x^4 - q^4, x^6 - q^6, x^4 + 4q^4, (x^l - q^l)/(x +- q), where l is even, and (ax^2 + bx + c)(dx + e), where ax^2+bx+c is irreducible. We also show that for x^l - q^l, with l congruent to 2 modulo 4, Fp(n) is not squarefull for infinitely many n. Our methods rely on techniques from Modular Arithmetic such as Hensel's lemma and the Lifting the Exponent lemma, as well as on concepts from Analytic Number Theory such as Partial Summation and results about distribution of primes in progressions.

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