General Theory of the Occurrence of Pi in the Mandelbrot Set and First Proof of Its Occurrence at the Parameter -3/4

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The occurrence of Pi in the Mandelbrot set is a phenomenon which has been known since its discovery in 1991. It has enjoyed great popularity among mathematicians ever since, even appearing on the YouTube Channel Numberphile and arousing the interest of hundreds of thousands of viewers. Despite being very popular and the long time since its discovery, the understanding of the phenomenon was very limited. The digits of Pi appear when counting iterations until a divergence benchmark is reached, for some points on the boundary of the Mandelbrot set. The occurrence of Pi at the point 1/4 has already been proven in 2001, but relied on trigonometric functions and could not greatly improve the intuitive understanding of this phenomenon due to its abstractness. In 2023 a geometric proof for the point 1/4 was presented by Paul Siewert, which motivated our work. Besides the point 1/4, the occurrence of Pi was examined at a few other points in the Mandelbrot set, namely -3/4 (where the phenomenon was originally discovered) and -5/4, though the origin of its occurrence has neither been understood nor proven. In this work, we aim to provide a general understanding of the occurrence of Pi and establish the first proof of its occurrence at the point of its original discovery -3/4. Through this work we hope to contribute to the general understanding of dynamic systems and chaos, which can present a theoretical foundation for future applications in areas such as climate research, cell biology or astrophysics.

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