Revisiting the Collatz Conjecture: Analysing Strings, Discovering Bounds, and Computing Distributions in Binary Collatz Orbits

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Lothar Collatz defined a function on natural numbers: divide x by 2 when it is even, multiply by three and add one otherwise. He conjectured that repeated application of this function to any natural number will result in 1. While this has remained unsolved for 87 years, a proof will have applications in blockchain and Shannon-perfect encryption schemes (theoretically more efficient than RSA). This research aimed to (a) derive a simplification of the conjecture, (b) analyze these modified orbits' binary representations, and (c) study distributions of Changes(x) vs Changes(C(x)). In my research, I have developed a novel inductive proof to show that the conjecture is equivalent to a repeated algorithm R (3x plus its largest dividing power of 2), with end states being powers of 2. Therefore, these values (and how R affects them) can be naturally analysed using binary. Secondly, I define a new function, “Changes”, as the number of adjacent unequal digit pairs (”01” or “10”) in a binary string. Performing relevant simulations, I proved that for all k-digit binary x, applying the Collatz function to x decreases Changes when Changes(x) exceeds |_2k/3 - 1/3_|, restricting growth of Changes; I proved that this is the best bound for all k. Additionally, I studied distributions plotting Changes(R(x)) against Changes(x), resulting in a predictable distribution for all k > 4. These theoretical and computational results take us towards a complete proof, also opening up a novel field of study in dynamical systems.

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