The Boltzmann-Hamel Equations for the Dynamics of Systems on Lie Groups and Lie Algebroids

Esquivel, Raphael (School: Iolani School)

We explore the Euler-Lagrange equations in which tangent spaces have non-coordinate bases. This generalized form is the Boltzmann-Hamel (BH) equation, which contain a term dependent on a coefficient defined as a structure function of the frame. A coordinate-free interpretation of Hamel coefficients enables a more intrinsic and geometrically invariant formulation of the equations of motion for mechanical systems on Lie groups and Lie algebroids. By leveraging the algebraic structures of SO(4) and SU(2), we systematically derive the Hamel coefficients without relying on explicit coordinate choices. Additionally, we extend the application of the Heisenberg group as a configuration space in Lagrangian dynamics, providing a counterpart to recent Hamiltonian treatments. The study further investigates the role of Lie algebroids, including Atiyah algebroids arising from principal bundles, in constrained Lagrangian system. We relate their geometric structure to the Boltzmann-Hamel equations using Lie algebroid connections. Applications to nonholonomic motion, modified Kepler problems on the Heisenberg group, and systems evolving on principal bundles illustrate the practical implications of these findings. This work broadens the scope of geometric mechanics by introducing new parameterization methods and extending classical mechanical formulations to more general Lie-theoretic settings.

Awards Won: