Make24: Elegant Elementary Bounds for a Numbers Game

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Make24 is a classic game where players compete to be the fastest to get to 24 using 4 integers between 1 and 10 inclusive. In the generalised version of this game, one is given k not necessarily distinct integers, each between 1 and 10 inclusive, and attempts to make a target number n (not necessarily 24). For fixed n, given k integers, a valid solution exists if one is able to use addition, subtraction, multiplication, division and parentheses to form an expression which evaluates to n where every operation yields an integer. For the case n = 24, it is well-known that for small k, like k = 4, the existence of a valid solution is not guaranteed. In this research, I define eta(n) for fixed n to be the minimum k which guarantees the existence of a valid solution and aim to compute values of eta. As a start, a novel algorithm to search for small values of eta and elementary identities to construct bounds for eta were developed. These results served as building blocks to create reduction steps to support a process of Strong Induction, and culminate in generalised bounds for eta(n). Empirically, eta exhibits peaks at primes, similar to functions like the related Mahler-Popken Integer Complexity, leading to intriguing prime-related observations, leading to potential contributions in Number Theory. Furthermore, the recursive nature of the algorithms involved brings potential applications in Algorithmic Analysis, and the game is also a way to train children's arithmetic skills and open their eyes to the beauty of mathematics. Most importantly, this game also exposes the layman to the intricate mathematical complexities which arise from simple rules, fascinating enthusiasts and amateurs alike as a fun brain teaser!

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