Slice Genus Bounds for Knots Using Grid Diagrams

Qiu, Christopher (School: Bridgewater-Raritan High School)

The smooth sliceness of a knot is an important property with connections to longstanding problems in topology such as the smooth four-dimensional Poincare Conjecture (4SPC) and the Slice-Ribbon Conjecture. Existing methods to determine the smooth sliceness of a knot use sophisticated algebraic tools such as knot homology and the Rasmussen s-invariant. We present a novel combinatorial methodology to lower bound the slice genus of a knot by studying its relationship with the grid index and the Thurston-Bennequin number, and provide a heuristic for which types of knots yield slice genus bounds high enough to guarantee smooth sliceness. In particular, our method recovers the slice genus bound for torus knots obtained by Kronheimer-Mrowka and Rasmussen. Additionally, we give a recursive construction for an infinite sequence of knots with slice genus approaching infinity. We identify a base knot 16n196836 that makes all knots in this sequence topologically slice. This concrete sequence of topologically but not smoothly slice knots is useful because potential counterexamples to 4SPC involve conditions on the sliceness of a knot that can be satisfied by the knots in this construction. Our construction demonstrates that the smooth sliceness of certain knots can be determined purely using combinatorial properties of its grid diagram.

Awards Won: