# Geometric Investigation of Quasicrystals: Analyzing Diffraction Patterns With de Bruijn's Multigrid Lattices

Roberto, Kyle Anthony (School: Valenzuela City School of Mathematics and Science)

Quasicrystals are materials with ordered but aperiodic atomic arrangements, which can be contrasted from traditional crystals that possess a periodic structure. Their unique thermal and electrical resistance, hardness, and non-stick properties have piqued industrial interest for their engineering applications. However, challenges in modeling and synthesis arise due to the complexity of their diffraction patterns and limited foundational research. This study reinterprets two centuries-old theories, previously overlooked by researchers, by applying classical Bravais lattices to map lattice points into tiling patterns, as described by multigrid lattice theory. These 2D tiling patterns provide insights into how atomic arrangements in quasicrystals may correspond to Bravais lattices, enhancing our understanding of their structure. This study proved that angles formed by intersecting lines in multigrid lattices occur in consecutive multiples of 180/n degrees. It also identified a new geometric relationship — that the ratio of the angles of intersection follow a similar formulaic manifestation of the Crystallographic Restriction Theorem, introducing the linkage of Crystals and Quasicrystals. Additionally, it was shown that n-folded multigrids could transform into 2n-folded ones, revealing a geometric equivalence between crystals and quasicrystals. Lastly, certain quasicrystals were found to have higher packing efficiencies than crystals, challenging established structural theories and providing a foundation for further research in material sciences. These geometric insights provide parameters for computational modeling, akin to Bragg's law for diffraction, which offers insights for advancing quasicrystal synthesis and applications in material science, optimization, and nanotechnology.

**Awards Won:**