Taylor Integration to Model the Dynamics of the Restricted Three-Body Problem

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This study explores the dynamics of the restricted three-body problem within the field of celestial mechanics by utilizing Taylor integration via programming in the Julia language. This extension of the three-body problem models the trajectory of a body of negligible mass under the influence of two massive bodies. Computationally, the computer program plots segments of n-order polynomials, whose coefficients are those of the Taylor series for such a system and appends them to create a singular curve. In this study, the Lagrange points are determined after imposing a constant ratio between the masses of two of the celestial bodies. Then, the Hill region and the zero velocity points corresponding to the energy level of a singular Lagrange point are graphically produced. Particular initial position and velocity values are chosen and the trajectory of the point mass is overlaid on the aforementioned graph. The model produced by the Taylor integrator was contrasted with a finite differencing technique frequently utilized in simulations within celestial mechanics. The Taylor integrator was shown to be at least 1000 times more accurate in the short run than the Finite-Differencing method and had an exponentially decreasing error over time. There was also no significant difference in the mean runtime of both programs with $p=0.05$. This decreasing error rate was also in part due to the implementation of a dynamic time step and change of coordinates. These results have uses in space exploration notably in improving the precision of flight-path calculations for spacecrafts. By accommodating the gravitational effects of a greater number of celestial bodies this approach has the potential to enhance fuel efficiency significantly resulting in lower mission expenses overall.

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