Solving Variable-Entangled Partial Differential Equations (PDEs) via Atomic Tensor Decomposition and Gated Expert Neural Networks: A Multi-Expansion Framework for Accurate and Adaptive PDE Approximation

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We propose a gated neural framework for solving partial differential equations (PDEs) of the form L(u(x)) = f(x), where x ? O ? Rn and u : O ? R. The solution u(x) is expressed as a residual-weighted sum of atomic separable expansions: u(x) ˜ S? w?(x) × ?? f??(x?), where each f?? captures univariate structure along dimension j. Atomic components are drawn from six novel analytically derived expansion families, each corresponding to a distinct structural mode of the PDE, including first-order, second-order, nonlinear coupling, fractional, eigenfunction, and PGD-based decompositions. For each atomic candidate f?, we compute a pointwise residual R?(x) = $|Lf? - f(x)|^2$. These residuals are passed to a gating network that assigns spatially adaptive weights using w?(x) ? exp(-R?(x)) / S? exp(-R?(x)), enabling the model to prioritize expansions that best match the local operator behavior. The PDE system, including L, f, and boundary data g, is embedded as a structured feature vector and fused with spatial coordinates. This representation is processed through six expert subnetworks, each generating a candidate solution aligned with a specific expansion class. The aggregated output û(x) maintains operator consistency, admits SVD-based low-rank compression, and remains interpretable. Unlike monolithic solvers such as PINNs, our model operates in the Fréchet space C^8(O), ensuring sup? |??u?/?x? - ??u/?x?| ? 0 ? k ? N via a hybrid soft-penalty/hard-solve loss, preserving termwise differentiability. This yields reduced approximation error in regions with steep gradients, faster residual decay ?L(u^) - f? ? 0, and consistent recovery of operator-aligned solutions in systems where ?u, u^2, or mixed derivatives $?^2u/?x?y$ dominate the solution behavior.

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