Solvability of Meromorphic Equations in Elementary Functions

Veselinov, Nikola (School: Sofia High School of Mathematics)

An equation $f(x)=a$, where f is a complex meromorphic function and a is a parameter, is solvable in elementary functions if the inverse map $x=f^{-1}(a)$ can be expressed as a finite composition of arithmetic operations, the exponential function, and the complex logarithm. Using methods from topological Galois theory, Kanel-Belov, Malistov and Zaytsev proved unsolvability for the equations $\tan x - x = a$ and $x^x = a$ (and its equivalent $\exp x + x = a$), while Zelenko covered almost all entire surjective functions of at most exponential growth. We generalize these results to prove that if the derivative of f has infinitely many roots x_i and the set of distinct values $f(x_i)$ is infinite, then $f(x)=a$ is unsolvable in elementary functions. The proof combines Wielandt's theorem on primitive actions of infinite permutation groups with an iterative decomposition of the monodromy group into primitive factors via one-dimensional topological Galois theory.

Awards Won:

First Award of $6,000

Regeneron Young Scientist Awards

American Mathematical Society: One-Year Membership to American Mathematical Society to each winner (7 winning projects, up to 3 team members per project)

American Mathematical Society: Second Award of $1,000

Mu Alpha Theta, National High School and Two-Year College Mathematics Honor Society: First Award of $ 1,500