The Split Elevation Theorem and the H Tower for Virtual Knots

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This paper investigates invariants for distinguishing virtual knots. The existing H_MJ polynomial suffers from a structural weakness: when its base polynomial evaluates to zero, essential crossing information is lost. To remedy this issue, we propose the Generalized H polynomial, which employs an ordered-pair structure to preserve the relevant information more faithfully. Building on this idea, we formulate and prove the "Split Elevation Theorem," a general mechanism that produces substantially stronger invariants from any given flat virtual knot invariant. Applying the theorem to established polynomials such as the Arrow polynomial, we construct enhanced invariants that distinguish knots previously indistinguishable by earlier tools. We also introduce a flatization procedure for the H invariant via an explicit equivalence relation. This allows H itself to be used recursively as an input to the Split Elevation Theorem, yielding an infinite hierarchy of progressively stronger invariants, which we call the "H tower" (H^n). Our computations indicate that higher-order invariants, such as H^2, can distinguish knots that lower-order invariants cannot, and they provide statistically tighter lower bounds for the crossing number.

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