An Investigation of Fractal Dimension Growth and Convergence in Mathematical Fractals Using Box-Counting and Perimeter Scaling Methods

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Fractals are geometric objects that are characterized by self-similarity and non-integer dimension. They appear in ideal mathematical constructions and in natural systems. This project examines how numerically estimated fractal dimension evolves with increasing iteration in mathematical fractals and compares this behavior to natural objects. The Koch snowflake and Sierpinski triangle were constructed through multiple iterations and analyzed using the box-counting method, where grids of decreasing box size e were overlaid and the number of intersecting boxes N(e) was recorded. Log–log plots of N(e) versus 1/e were used to estimate fractal dimension from the slope of the linear region. To validate the results, an independent perimeter-scaling method was applied to the fractals. The results show that increasing iteration leads to higher measured fractal dimensions that converge toward theoretical values. Natural objects showed approximate linear scaling over a limited range of e but deviated at extreme scales due to physical structure and image resolution constraints. These findings support the hypothesis that mathematical fractals demonstrate convergence of fractal dimension with iteration, while natural systems display scale-limited fractal behavior. This study highlights the importance of scale selection, measurement uncertainty, and methodological consistency when analyzing fractal geometry in idealized and real-world systems.

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