Sampling the Complete Configuration Space of Origami and Linkages Using Markov Chain Monte Carlo

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Origami and linkages are widely used across architecture, space engineering, and the natural world owing to their high interconnectivity, where a single input drives the entire mechanism. Both exhibit multiple modes of motion separated by rigid and non-rigid (snap-through) transitions. Existing methods—kinematic approaches based on geometric constraints and mechanics-based approaches minimizing elastic energy—trace only a single mode per run or become trapped in local minima, preventing efficient exploration of the full configuration space. This work presents a unified framework that samples the complete configuration space in a single run by leveraging an analogy with statistical mechanics. Both systems are represented as graphs, with joints/vertices as nodes and bars/creases as edges. An artificial energy defined as the squared deviation of edge lengths from their original values makes rigid configurations correspond to low-energy, high-probability states under a Boltzmann distribution. This distribution is sampled via Markov Chain Monte Carlo, combining the Metropolis algorithm, Hamiltonian Monte Carlo, and parallel tempering to explore within modes and across energy barriers. Validation against known analytical solutions shows exact agreement: spectral clustering identifies four modes for a four-bar linkage and five for a single-vertex origami. For ladybug wing folding, three clusters emerge, providing evidence that the diamond-shaped crease pattern induces strong bistability. The framework also extends to inverse design, estimating linkage parameters that generate trajectories through specified points. By bridging statistical mechanics and kinematics, it opens new avenues for analyzing complex mechanisms in both engineered systems and natural phenomena.

Awards Won:

First Award of $6,000

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