Projective Generalization of Foci and Isogonal Cubics Loci of Complete Quadrilaterals

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The theory of isogonal cubics is a powerful tool in proving the relations of angles. A part of the theory is deduced from Newton's theorems on conics. It is known that Newton's theorems can be generalized, so it's a natural question to ask if the entire theory of isogonal cubics can be generalized. In this study, we used projective geometry to describe and deduce properties of conics, which gave a whole new perspective to the Euclidean definition of conics and related objects. We start from generalizing fundamental geometrical objects such as the foci of conics, pedal circles and Miquel points projectively. Motivated by these generalizations, we proved that for a point, there exist an isoconjugate of it regarding the complete quadraliteral if and only if it satisfies a certain cross ratio equation. Finally, we proved that an isoconjugation is equivalent to an isoconjugation on a specific triangle, we used this result to prove the locus of isoconjugation is a cubic.

Awards Won:
Fourth Award of $600