Fractal-Type Structures Associated With the Riemann Xi Function

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An infinite iterated function system (IFS) associated with local inverse branches of the Riemann xi-function near its simple zeros is constructed. Throughout, all zeros of the xi-function are assumed to be simple, consistent with all zeros computed to date. After explicit affine normalization, this system generates an invariant set K exhibiting recursive self-similar features characteristic of fractal-type geometry. Although the zero set itself is countable, the associated invariant set is uncountable. Detailed analytic proofs of the local inverse construction, contraction properties, and existence and uniqueness of K via the Banach fixed-point theorem are given. We further prove that the functional equation ?(s) = ?(1 - s) induces an antipodal symmetry on K, a geometric property that is specific to the xi-function and absent for generic entire functions. Hausdorff dimension is studied using pressure methods under explicit summability assumptions. A toy model on the unit interval illustrates the mechanism independently of the xi-function.

Awards Won:
American Mathematical Society: Honorable Mention and One-Year Membership to AMS (for 5 projects with up to 3 team members per project)