# Complexity Functions Are All You Need

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What does a resistor network, the popular British game show Countdown, and a function from number theory have in common? These seemingly unrelated concepts share a fundamental, mathematical structure. From mathematics to engineering, many problems require the construction of a target object from pre-defined constituents, just as one builds a castle from Lego bricks. The first part of this research introduces the mathematical framework of Formation Complexity Functions (FCFs), designed to identify an efficient way to assemble complex objects. A wide range of problems, spanning number games in combinatorics, integer complexity in number theory, and resistor networks in engineering, can be described within this framework using only five parameters. An important parameter in the definition of an FCF is the set of legal operations one can use to combine the building blocks to create new objects. By studying certain sets of legal operations, some mathematical structure and properties of the FCFs can already be determined. The generality of such results provides insight into the asymptotic growth of various classes of FCFs. This paper culminates in the study of a specific number game described by FCFs. Roll k = 4 fair dice with m = 6 sides. If one tries to use the numbers rolled exactly once each to form a mathematical expression evaluating to n = 24, only using parentheses, addition, subtraction, multiplication and division, the probability of success is well-known to be less than unity. A natural question to ask is therefore: how large must the number of dice, k, be, to guarantee success? This paper answers this question with logarithmic upper bounds for any general target number n, and further generalises to dice with any number of sides, m.

**Awards Won:**
Third Award of $1,200
American Mathematical Society: Honorable Mention and One-Year Membership to AMS (for 5 projects with up to 3 team members per project)