A Generalized Theory of Divisor Topologies in the Framework of Tau-Factorization

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This research aims to characterize divisor topologies within the framework of tau-factorization where tau is a symmetric relation on the set of non-zero, non-unit elements of an integral domain. The research will follow the open set construction outlined by Yigit and Koc(2024). Their idea considers building and understanding the set of factors of each non-zero, non-unit element of a commutative ring. In this case, the methods will be applied to the sets of tau-factors of the elements on a Unique Factorization Domain. These sets of tau-factors act as basis elements to construct the open sets of the tau-divisor topology on an integral domain. The construction of basis elements was analyzed under 3 distinct types of relations: divisive relations, associate preserving and refinable relations, and relations in which the tau division operator is transitive. Examples, characterizations, and subsequent topological properties for each of the three cases are analyzed and discussed. Any relation in which the tau-divide operator is transitive forms a tau-divisor topology which is a Kolmogorov topological space. Moreover, when tau is a divisive relation on a Unique Factorization Domain, the tau-divisor topology formed is an Alexandrov space. Under this framework, the notion of tau-divisor topologies was generalized and a new family of Alexandrov topologies was developed.

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