A Novel Spectral-Stochastic Mathematical Framework for Water Scarcity Mitigation via Physics-Informed Neural Operators

Rastogi, Advika (School: duPont Manual High School)

Globally, water scarcity affects 75% of the population. Agriculture accounts for 70% of global freshwater withdrawals, and farmers use unreliable, reactive irrigation systems. This project introduces a novel three-component framework: Fourier Neural Operator (FNO), Stochastic Differential Equation Extension (SDE), and Model Predictive Controller (MPC). The FNO models soil moisture dynamics with an L^2 error <0.01, 100,000 parameters, and a confirmed power-law convergence rate (slope=-1.34, R^2=0.927). The SDE adds a learned noise term to the Richards equation and performs uncertainty quantification, with 97.3% of spatial locations satisfying theoretical bounds and a KL divergence of 0.047 against the reference distribution. The MPC transforms these forecasts into irrigation decisions, enforcing a 95% probabilistic soil-moisture safety constraint over a 7-day receding-horizon, achieving Lyapunov stability and zero constraint violations across 1,000 stress-test simulations. At 500 hectares, the MPC yields $99M and 600M m^3 of water saved. Hardware validation of 20 plants (n=10) yielded a statistically significant (p<0.0001) 44.7% reduction in water use with no statistically significant effect on plant health. Computational global validation across 50 sites on six continents produced an average water savings of 22.3% and 100% success rate. Scaled globally, this framework stabilizes food production for 32 million people and conserves 60 billion m^3 of water. From the framework, three theorems are produced: a suboptimality bound for MPC under absolute operator error, a minimum stabilizing horizon formula N*(e), and a KL divergence bound under spatially correlated learned noise.

Awards Won:
American Mathematical Society: One-Year Membership to American Mathematical Society to each winner (7 winning projects, up to 3 team members per project)
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