Computational and Statistical Analysis Maximum Growth in the Collatz Conjecture: Evidence of a Scaling Law and Local Arithmetic Structure

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The purpose of this study is to investigate the maximum growth M(n) of Collatz orbits and to propose that its variability is structured rather than random. A total of 832,000 values were analyzed using stratified sampling. The analysis was implemented in Python with a reproducible computational pipeline and weighted inference to ensure unbiased estimates and statistical reliability, as well as balanced coverage across arithmetic classes. A power-law model M(n) = an^b was used to describe the global trend, together with a structural residual that quantifies deviations from expected growth. The results show that maximum growth follows a stable scaling law with an exponent close to 1 (b estimated as 1.000469), consistent across multiple partitions of the domain. The residual exhibits non-Gaussian behavior, with positive skewness (1.37), elevated kurtosis (3.56) and a strong deviation under normality tests, which indicates that extreme growth events occur more frequently than expected under random noise. Tail analysis based on exceedances favors a lognormal distribution over Pareto using the AIC criterion, supporting structured extreme growth. Systematic differences were identified across arithmetic classes, with large effect sizes (Cliff's delta between 0.56 and 0.85) and high statistical power. A positive relationship was observed between the residual and stopping time, linking extreme growth with trajectory complexity. These results support a hierarchical model with global scaling and a structured residual linked to arithmetic properties.

Awards Won:

American Mathematical Society: Honorable Mention and One-Year Membership to AMS (for 5 projects with up to 3 team members per project)