On Scale Dependent Velocity Fluctuations Generated by Molecular Collisions in Coarse-Grained Fluid Motion as a First Principles Basis for Stochastic Hydrodynamics and Their Implications for Navier-Stokes Smoothness Through Scale Time Limitations of Continuum Averaging

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This project develops a first principles framework linking molecular-scale randomness to macroscopic fluid behavior through coarse-grained velocity fields. Modeling molecular collisions as a discrete stochastic process, a binomial random-walk formulation yields closed-form expressions for velocity variance as a function of time and averaging scale. The analysis shows that collision-induced fluctuations persist under finite coarse-graining and decay as a power law in averaging length, establishing a quantitative bridge between microscopic dynamics and continuum descriptions. This result provides a physical basis for stochastic hydrodynamic models, particularly the Landau–Lifshitz formulation of the Navier–Stokes equations. Rather than introducing noise phenomenologically, the derived scaling law enables direct calibration of stochastic forcing from collision statistics. This supports more physically grounded computational fluid dynamics, especially in microfluidic and mesoscale regimes, where finite particle effects are significant. A second contribution is a reinterpretation of the Navier–Stokes existence and smoothness problem. The results reveal a scale–time incompatibility: as averaging scale decreases, velocity variance grows over time, preventing uniformly bounded smooth fields across arbitrarily small scales and long times. This suggests smoothness is an emergent consequence of averaging over many particles. From this perspective, potential singularities can be interpreted as a breakdown of the coarse-grained description (sampling collapse) rather than true physical divergence. This work connects stochastic molecular dynamics with continuum theory and highlights the need for stochastic or multiscale models in regimes where classical assumptions fail.

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