Energy-Stable Numerical Method for Blood Flow in 3D Brain Aneurysms

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Brain aneurysms are balloon-like bulges in brain arteries, where abnormal blood flow is believed to influence whether an aneurysm grows or ruptures. This project studies a mathematical model of blood flow in 3D aneurysms using numerical simulations. Blood is treated as an incompressible, shear-thinning fluid described by the Carreau model, where viscosity depends on the local shear rate, leading to a nonlinear version of the Navier–Stokes equations.Many existing models assume constant viscosity, use low-order time methods, or lack strong stability guarantees. To address this, I developed a finite element–based numerical method that is second-order accurate in both time and space. The method uses a pressure-correction approach, splitting the computation into velocity and pressure steps.A key feature of the method is energy stability: it satisfies a discrete energy law, ensuring that the numerical energy does not increase over time, regardless of time step or mesh size. This provides a strong guarantee that the simulation remains stable even in complex 3D geometries. Numerical tests confirm second-order convergence and proper energy dissipation. The method is then applied to patient-specific aneurysm geometries reconstructed from medical data. The simulations reveal detailed flow patterns, including jets and recirculation zones inside the aneurysm sac, which are difficult to measure directly in patients.Overall, this work provides a stable and accurate computational approach for simulating realistic blood flow in aneurysms and offers a foundation for future patient-specific studies of aneurysm growth and rupture risk.

Awards Won:

Fourth Award of $600

Mu Alpha Theta, National High School and Two-Year College Mathematics Honor Society: First Award of $ 1,500