The Yarlagadda Criterion: Quantifying Relativistic Deformation of Chaotic Phase Boundaries in the Three-Body Problem for High-Precision Orbital Stability and Deep-Space Navigation

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As the discovery of compact multi-planet systems advances, classical Newtonian dynamics are proving insufficient for predicting long-term orbital stability, resulting in over-optimistic orbital models that could jeopardize architectures of observed exoplanetary systems. While relativistic influences are often considered "negligible" in planetary regimes, this research demonstrates that first-order post-Newtonian (1PN) corrections act as fundamental "boundary shifters" in chaotic systems. To address this gap, a comparative analysis was conducted between Newtonian and 1PN dynamical frameworks utilizing the high-accuracy IAS15 integrator within the REBOUND N-body framework. Over 10,000 synthetic systems were simulated under controlled variations in mass ratios, eccentricities, and orbital spacing to quantify the 1PN-induced deformation of chaotic phase-space boundaries. The results show that relativistic periapse precession systematically triggers resonance drift and unpredictably shifts stability zones. To quantify this transition, the Yarlagadda Criterion (Xrel) was developed: a mathematical threshold defined as Xrel = 1PN Precession Rate/ Synodic Frequency. Using Xrel, significant chaos was observed at a calculated dimensionless threshold of Xrel = 0.00376, where exceeding 0.00376 redefines the stable habitability zones. Stability models were then validated using Mean Exponential Growth of Nearby Orbits (MEGNO) and Lyapunov indicators over 10^6 orbital periods, confirming that 1PN corrections alter divergence rates by shifting Mean-Motion Resonance (MMR) locations. Structural analysis of the generated phase maps proves that these shifts are timing-dependent, establishing the Yarlagadda Criterion as a required analytical tool for high-precision orbital modeling.

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