The Repelling Propellers Problem: Optimizing Energy Configurations in Orbital Patterns

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This project focuses on the Repelling Propellers Problem, an unsolved problem proposed by Professor James Propp. The question investigates how same-sign, equally charged/spaced points behave when confined to multi-dimensional spaces (circles, spheres) and allowed to reach equilibrium under the Coulomb log-gas potential. A 2D version of the problem is multiple propellers on a single spindle, where the ends of the propellers are charged to "repel" each other, and the goal is to determine the angles at which the propellers will find equilibrium. As this problem generalizes, there are many applications: sphere packing, satellite placement, electrochemical surface science, wireless networks, numerical integration, crystallography, vibration/oscillation science, and protein formation. The project methodology started by creating a physical and digital model to get an intuition for patterns. Using complex-number and cyclotomic techniques, the closed-form formula for two-propellers in 2D was proved in terms of the gcd, yielding an exact, verifiable minimum. The three-propeller case was reduced to maximizing a sine product, allowing for classification of all propellers as "good" (achieve absolute minimal energy), "kind" (optimized at rational angles), or "bad" (neither good nor kind). For "kind" sets, the project develops the Tangent Ratio Conjecture by restricting rational tangent diophantine equations, proving it via two independent approaches (cyclotomic algebra and algebraic-degree arguments). In 3D, the work defines planar propellers and charged regular polyhedra, identifying rigid equilibrium behavior and characterizing global-energy-minimizing configurations. The project finds the closed-form formula for two and three propellers in 3D.

Awards Won:
Third Award of $1,200
American Mathematical Society: One-Year Membership to American Mathematical Society to each winner (7 winning projects, up to 3 team members per project)
American Mathematical Society: First Award of $2,000