A Class of Convex Curves Arising in Capillary Floating Problem

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Capillary floating problem in physics concerns the equilibrium positions of floating bodies whose positions and orientations are governed by capillary forces. It plays an important role in manufacturing colloidal nanoparticles. To reconstruct the shape of a body from its projection function is a crucial technical step in imageology. Motivated by capillary floating problem and billiard ball problem in mathematics, we introduce a new projection function for convex plane curves, and study the curves when the defined projection function is constant. We call such curves as curves of constant projection (CCSP). We find that the projection function can be expressed in convolution form involving the radius of curvature. Using Fourier expansion technique, we give a necessary and sufficient condition for the existence of CCSP curves, and provide an explicit expression for the radius of curvature. The existence is determined by a set of trigonometric equations. We also discussed the properties of CCSP curves and their relation to Gutkin curves from billiard ball problem. Finally, we construct and exhibit some curves with constant projection function. CCSP curves can be regarded as a generalization of curves of constant width. They have great potential applications in engineering design.

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