On the Effective Hilbert Zero Point Theorem

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It is a fundamental problem to find a good criterion to decide whether a system of polynomial equations has a solution. Hilbert Zero Point Theorem says that for two polynomials f(x,y) and g(x,y) without common zero points over complex number field, there are two Bezout polynomials u(x,y) and v(x,y) such that f(x,y)u(x,y)+g(x,y)v(x,y)=1. (1) This gives a theoretic criterion for two polynomials admitting a common complex zero point. In order to get an effective criterion, J. Kollar got in 1988 an Effective Hilbert Zero Point Theorem, namely, he gave some upper bounds on the degrees of the Bezout polynomials. Kollar's theorem reduces the existence problem of u and v to the solution of a system of linear equations. Our purpose is to give a simple criterion without solving linear equations, which implies also the effective Hilbert zero point theorem. The main tool of our method is the resultant of two polynomials. The main result of my project is to give formulas to compute the Bezout polynomials in Hilbert Zero Point Theorem. As an application, we get a simple criterion for the existence of common zero points of two polynomials. The new criterion is better than Kollar's, we do not need to solve linear equations. We will also try to use our method to study some open problems on polynomials.

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