Generalized Problem of Apollonius

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Problem of Apollonius is a famous geometric problem. Apollonius of Perga posed it in the 3d century BC. The statement of the problem: construct (by straightedge and compass) a circle that is tangent to three given circles in a plane. It is a well-known fact that this problem has at most eight real solutions (we exclude the degenerate case of all the given circles are tangent to each other at the same point). My research is about one of the generalizations of this problem. I found a maximal possible number of solutions in case of more than the three given circles. Using methods of elementary geometry, I proved that in case of the four given circles there exist at most six solutions and in case of the five given circles there exist at most four solutions. Case of more than the five given circles is a simple corollary of the proved theorems. After finding the maximal possible number of solutions in case of the four given circles, it is possible to classify all quaternaries of circles that have exactly six solutions. It turns out that using inversions and similarities (so called Mobius transformations) every such quaternary could be reduced to one of the four canonical forms. Two of these forms depend from two parameters and two other forms do not depend from any parameters, so they are “rigid”.

Awards Won:

Fourth Award of $500

American Mathematical Society: Third Award of $500