Polynomials in Z[x] and Irrationality Measure

Macris, Griffin

The field of Diophantine approximation is a branch of number theory which concerns itself with certain properties of real numbers in relation to the rational numbers. In specific, one may define the irrationality measure of a real number to give a meaning to how “close” it is to the surrounding rationals. Several important theorems have been proved recently, including the determination of the irrationality measure of all algebraic numbers. I hypothesized that I would be able to prove a relationship between the irrationality measure of a number x, and the irrationality measure of a polynomial P(x). In addition, I wanted to find out if I could do so without the use of calculus or analysis, as many concise proofs of theorems in the field of Diophantine approximation rely on deep theorems from these areas. I present a novel theorem which relates the irrationality measure of a real number x to the irrationality measure of the real number P(x), whenever P is a polynomial with rational zeros. The proof shows that the irrationality measure of P(x) is necessarily bounded below by a function of the irrationality measure of x. I prove this relation using algebraic methods and induction. I then extend this proof to complex numbers. This is accomplished by replacing the real absolute value with the complex norm in the proof. I show that the same relation holds, wherein the polynomial P has roots which have rational real and imaginary parts.

Awards Won:

American Mathematical Society: First Award of $2,000