Orthogonal Polynomials and the Two Dimensional Nevai Condition

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This work deals with an open question in math with direct implications to spectral theory and to mathematical physics. A measure describes the distribution of mass over a general set. Given a measure, one can define the integral of a function against it as the weighted average of this function, weighted according to this measure. (Two simple examples would be one where the total mass of the measure is concentrated on a single point, in which case the integral of a function is simply its value at the point and one where the mass of the measure is uniformly distributed over an interval, in which case the integral reduces to the usual notion of integral of a function) Let m be a measure on the real line so that all of its mass is concentrated on a bounded interval. We also assume that the total mass of the interval is 1. Fix a point, y, on the real line and consider the polynomials of degree n which have the value 1 at y. From all those polynomials, we chose the one such that the integral, against m, of its squared value, gets the minimal possible value. We denote it Qn. Thus, the integral of Qn^2(x) is minimal among integrals of p^2(x) where deg p < n+1 and p(y)=1. Note that the only requirement preventing Qn from being identically 0 is Qn(y)=1. It is reasonable to expect Qn to be very small far away from y, and close to 1 in a neighborhood of y as n grows. This will imply that most of the mass of Qn^2 should be concentrated in a neighborhood of y as n grows. If this indeed holds at y, we say that m satisfies the Nevai condition at the point y. My research question is “does the Nevai condition hold for any measure satisfying the conditions above almost everywhere?” In this project, I have proven that a two-dimensional version of this condition always holds.

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