n-Dimensional Fractions and a Generalized Calkin-Wilf Tree

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This paper provides a generalization of the Calkin-Wilf tree. The Calkin-Wilf tree is an infinite binary tree in which all nodes are labeled by reduced positive rationals and each reduced positive rational labels exactly one node. An n-dimensional fraction over Z is a formal symbol a_1/a_2/.../a_n/b where a_i,0\neq b \in Z, is in reduced from if and only if gcd(a_1,a_2,...,a_n,b) = 1, and is positive if and only if a_i,b > 0. The purpose of this research project is to construct and investigate the graph structure of the n-dimensional Calkin-Wilf tree which has the following property: all nodes are labeled by reduced positive n-dimensional fractions and each reduced positive n-dimensional fraction labels exactly one node. The n-dimensional Calkin-Wilf tree is constructed by the reverse Euclidean algorithm. The graph structure of the n-dimensional Calkin-Wilf tree is determined completely by Diagram(n+1) which is a finite directed graph in which all nodes are labeled by partitions of n+1, each partition of n+1 labels exactly one node, and each edge is labeled by a certain positive integer.

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