

n-Dimensional Fractions and a Generalized Calkin-Wilf Tree

Yamin, Ajmain

This paper provides a generalization of the Calkin-Wilf tree. The Calkin-Wilf tree is an infinite binary tree in which all nodes are labeled by reduced positive rationals and each reduced positive rational labels exactly one node. An n -dimensional fraction over $\mathbb{Z}$ is a formal symbol $a_1/a_2/\dots/a_n/b$ where $a_i, 0 \neq b \in \mathbb{Z}$, is in reduced form if and only if $\gcd(a_1, a_2, \dots, a_n, b) = 1$, and is positive if and only if $a_i, b > 0$. The purpose of this research project is to construct and investigate the graph structure of the n -dimensional Calkin-Wilf tree which has the following property: all nodes are labeled by reduced positive n -dimensional fractions and each reduced positive n -dimensional fraction labels exactly one node. The n -dimensional Calkin-Wilf tree is constructed by the reverse Euclidean algorithm. The graph structure of the n -dimensional Calkin-Wilf tree is determined completely by $\text{Diagram}(n+1)$ which is a finite directed graph in which all nodes are labeled by partitions of $n+1$, each partition of $n+1$ labels exactly one node, and each edge is labeled by a certain positive integer.

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