The Homotopy Theory of Parametrized Objects

Yegnesh, Karthik

In their study of generalized covering maps in categorical Galois theory, Borceux and Janelidze analyzed topological properties of the coproduct completions of small categories. Up to equivalence, the coproduct completion Fam(C) of a small category C is the category whose objects are objects of C indexed over arbitrary sets and whose morphisms are induced by morphisms in C. They showed that certain categorical properties of objects in Fam(C) are determined by the “geometric” properties of the sets that parametrize them. When C is not an ordinary category, but rather an ∞-category, objects in Fam(C) are objects of C parametrized by homotopy types (or ∞-groupoids), not just sets. This gives evidence for a novel and fruitful connection between the homotopy theory of spaces and colimit completions of higher categories, resulting in powerful applications of geometric and topological intuition to higher category theory. In this work, we develop this connection by studying the homotopy theory of such “parametrized families” of objects in ∞-categories as generalization of the classical homotopy theory of spaces. In particular, we study homotopy-theoretical constructions that arise from the fundamental ∞-groupoids of families in an ∞-category. In the same spirit, we show that for any ∞-category C, Fam(C) admits a Grothendieck topology which generalizes Carchedi’s canonical Grothendieck topology on ∞-topoi, thus allowing parametrized families of objects in C to be technically treated as open sets of a topological space. These results make a precise and powerful connection between data in higher categories and topological spaces, allowing for new applications of topological techniques to the study of higher categories.

Awards Won:

Intel ISEF Best of Category Award of $5,000

Intel Foundation Cultural and Scientific Visit to China Award